# Advantages of a Fuzzy-Paraconsitent Approach to Vagueness

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Aj's New Functors

| p | Weak Negation<br>p is false<br>~p | Strong Negation<br>p is totally false<br>¬p | Weak Affirmation<br>p is more or less true<br>Lp | Strong Affirmation<br>p is totally true<br>Hp |
|---|---|---|---|---|
| +1 | 0 | 0 | 1 | 1 |
| ± ¾ | ¼ | 0 | 1 | 0 |
| ± ½ | ½ | 0 | 1 | 0 |
| ± ¼ | ¾ | 0 | 1 | 0 |
| - O | 1 | 1 | 0 | 0 |

/~p/ = 1 - /p/

$/\neg p/ = \begin{cases} 1, \text{ if } /p/ = 0 \\ 0, \text{ otherwise} \end{cases}$

$/Lp/ = \begin{cases} 1, \text{ if } /p/ > 0, \\ 0, \text{ otherwise} \end{cases}$

$/Hp/ = \begin{cases} 1, \text{ if } /p/ = 1, \\ 0, \text{ otherwise.} \end{cases}$

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Simple Conditional

| ⊃ | 1 | ¾ | ½ | ¼ | 0 |
|---|---|---|---|---|---|
| 1 | 1 | ¾ | ½ | ¼ | 0 |
| ¾ | 1 | ¾ | ½ | ¼ | 0 |
| ½ | 1 | ¾ | ½ | ¼ | 0 |
| ¼ | 1 | ¾ | ½ | ¼ | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |

Implication

| → | 1 | ¾ | ½ | ¼ | 0 |
|---|---|---|---|---|---|
| 1 | ½ | 0 | 0 | 0 | 0 |
| ¾ | ½ | ½ | 0 | 0 | 0 |
| ½ | ½ | ½ | ½ | 0 | 0 |
| ¼ | ½ | ½ | ½ | ½ | 0 |
| 0 | ½ | ½ | ½ | ½ | ½ |

Biconditional

| ≡ | 1 | ¾ | ½ | ¼ | 0 |
|---|---|---|---|---|---|
| 1 | 1 | ¾ | ½ | ¼ | 0 |
| ¾ | ¾ | ¾ | ½ | ¼ | 0 |
| ½ | ½ | ½ | ½ | ¼ | 0 |
| ¼ | ¼ | ¼ | ¼ | ¼ | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 |

Strict Equivalence

| ↔ | 1 | ¾ | ½ | ¼ | 0 |
|---|---|---|---|---|---|
| 1 | ½ | 0 | 0 | 0 | 0 |
| ¾ | 0 | ½ | 0 | 0 | 0 |
| ½ | 0 | 0 | ½ | 0 | 0 |
| ¼ | 0 | 0 | 0 | ½ | 0 |
| 0 | 0 | 0 | 0 | 0 | ½ |

$/p \supset q/ = \neg p \vee q$

$/p \supset q/ = \begin{cases} 1, \text{ if } /p/ = 0 \\ /q/, \text{ otherwise.} \end{cases}$

$/p \rightarrow q/ = \begin{cases} \tfrac{1}{2}, \text{ if } /p/ \leq /q/ \\ 0, \text{ otherwise.} \end{cases}$

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| **Tautology?** | ~ | ¬ |
|---|---|---|
| N0 ↔ 1 | ✓ | ✓ |
| N1 ↔ 0 | ✓ | ✓ |
| p∨Np | ✓ | ✓ |
| N(p∧Np) | ✓ | ✓ |
| p ≡ NNp | ✓ | ✓ |
| p→q ⊃. Nq→Np | ✓ | ✓ |

| **Tautology?** | ~ | ¬ |
|---|---|---|
| p∧Np ⊃ q | ✗ | ✓ |
| p∨q∧Np ⊃ q | ✗ | ✓ |
| p⊃q ⊃. Nq⊃Np | ✗ | ✓ |
| Np ↔ NLp | ✗ | ✓ |
| p ↔ NNp | ✓ | ✗ |
| ½→. p∨Np | ✓ | ✗ |
| p∨q ↔ N(Np∧Nq) | ✓ | ✗ |
| p∧q ↔ N(Np∨Nq) | ✓ | ✗ |
| N½ ↔ ½ | ✓ | ✗ |

Soritical series

Ordered collection of elements differing in Ψ.
Ψ: underlying quantitative dimension, on which F supervenes.
Ψ orders the members.
(1) Two extremes: $a_0$ is F, and $a_z$ is not F.
(2) $a_i$ and $a_{i+1}$ are similar and dissimilar.

Is the soritical series possible?
Are (1) and (2) compatible?

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Soritical series

How to capture (2)?

(Similarity P.) $Fa_i \wedge Fa_{i+1} \vee . \sim Fa_i \wedge \sim Fa_{i+1}$
Fairness: Treat similar cases similarly
(Continuity P.) $\sim(Fa_i \wedge \sim Fa_{i+1})$
(Parity P.) $\sim Fa_i \vee Fa_{i+1}$

(Preservation P.) $Fa_i \supset Fa_{i+1}$
Completely false, for bounded series.
CL unable to accommodate (2).

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The Transition Questions

Q1: Its Nature: How?
Q2: Its Cause: Why?

The Problem

If there is a transition, contradiction.
$a_{50}$ is F, and not F

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*Discontinuism*

Soritical Series impossible.
Continuity Principle: $\sim(Fa_i \wedge \sim Fa_{i+1})$, is false.
DiscontinuityThesis: $\exists a_i, a_{i+1}\, (HFa_i \wedge \neg Fa_{i+1})$.
Sharp boundary: $a_i$ bipartitions the series.
There are no proper borderline cases.
Answer to Q1: Sudden transition.
Answer to Q2: Passage from $a_i$ to $a_{i+1}$ accounts for transition.

*Discontinuism*.- Assessment

Change reduced to a precipitous replacement.
Transitions are contradictory.
Example: Walking out of the room.
Reductio ad absurdum not valid for ~.
Small changes in Ψ ⇨ large changes in F.
Change not continuous.
Change in F not explained by change in Ψ.
Cut-off point, $a_i$, is arbitrary.
Sharp boundary is unfair, and against likeness.

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The Slippery Slope

Hypothesis: Z is absolutely not close to A
B is close to A
C is close to B
∴ C is close to A
D is close to C
∴ D is close to A
⋮
∴ Z is close to A.

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*Maximalism*

Rejects the rule of inference: transitivity.
For x to be F, it needs to be absolutely F.
Only prototypes.
What is good? The optimum.
A sentence is true only if totally true.
p ⊢ Hp.

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*Maximalism*.- Assessment

Deficient instances eliminated.
Fuzziness (= intermediate cases) abolished.
Massive impoverishment of reality.
No degrees and no comparatives.
Far too demanding.
Accept conclusion: ∀x (Fx).
But this is 1/∞ true.
∃x~(Fx) is 1/∞ false.

*Contradictorial Gradualism*

Fuzziness: intermediate zone in soritical series.
Gradual and contradictory.

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Degrees of Properties

If $a_i$ and $a_{i+1}$ were F to same extent, F unceasing.
If no degrees of F, no smooth change.
If x is less F than y, and y is less F than z, there are degrees.

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Degrees of Truth

From degrees of property to degrees of truth.
Redundancy Truth: 'a is F' is true $\leftrightarrow$ a is F.
The right member is gradual.
$\therefore$ The left member is gradual.
Generalization: 'a is F' is ... true $\leftrightarrow$ a is ... F.
Degrees of truth designated and antidesignated.

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*Minimalism*:

p is more or less true $\vdash$ p is true.

Acquiescence Rule: Lp $\vdash$ p.

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From Degrees to Contradiction

Something not totally F is partially not F.

$Lp \wedge {\sim}Hp \supset L\ {\sim}p$.

$Lp \wedge L{\sim}p$.

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Gradual Transition

Inverse Covariance of Opposites:
F diminishes as not-F augments.
$a_{50}$ is a soft limit.
There is no discontinuity.
Continuity, ${\sim}(Fa_i \wedge {\sim}Fa_{i+1})$, preserved.
Its truth ranges from 0.5 to 0.99.
True and false.
Sorites avoided: DS for ~ is invalid.

Advantages of Fuzzy-Paraconsistent System *Aj*

Soritical series is real.
There are intermediate situations
*Aj* is a strict extension of CL.
Keeps ⟨ weak bivalence: Every “p”: true or false.
continuity and fairness.
Avoids ⟨ discontinuism: smooth transition.
maximalism.
Speakers lack of agreement ⇨ contradiction.

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Advantages of Fuzzy-Paraconsistent System Aj

The PEM is (partially) true and (partially) false.
Reconciliation of agnosticism and indeterminism.

| | Excluded Middle | Non Contradiction |
|---|---|---|
| Simple, or Weak | (1) p∨~p<br>(2) Lp∨¬p | (6) ~(p∧~p)<br>(7) ¬(Lp∧¬p) |
| Strong | (3) p∨¬p | (8) ¬(p∧¬p) |
| Absolute | (4) H(p∨~p)<br>(5) Hp∨¬p | (9) H~(p∧~p)<br>(10) ¬(p∧~p) |

(1)

| p | ∨ | ~p |
|---|---|---|
| +1 | 1 | 0 |
| ±¾ | ¾ | ¼ |
| ±½ | ½ | ½ |
| ±¼ | ¾ | ¾ |
| -0 | 1 | 1 |

(2)

| Lp | ∨ | ¬p |
|---|---|---|
| 1 | 1 | 0 |
| 1 | 1 | 0 |
| 1 | 1 | 0 |
| 1 | 1 | 0 |
| 0 | 1 | 1 |

(3)

| p | ∨ | ¬p |
|---|---|---|
| 1 | 1 | 0 |
| ¾ | ¾ | 0 |
| ½ | ½ | 0 |
| ¼ | ¼ | 0 |
| 0 | 1 | 1 |

(4)

| H | (p∨~p) |
|---|---|
| 1 | 1 |
| 0 | ¾ |
| 0 | ½ |
| 0 | ¾ |
| 1 | 1 |

(5)

| Hp | ∨ | ¬p |
|---|---|---|
| 1 | 1 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 1 | 1 |